Additive combinatorics

Winter semester 2016/2017

Series VI

6.1. Suppose that $A \subseteq [N]$ and $|A| > 4N^{2/3}$. Show that there exists $d \in A - A$ such that $d/2, d/3 \in A - A$.

6.2. Let $A \subseteq \mathbb{Z}_p$, $|A| = \delta p$, where δ is a constant. Suppose that A shares an element with every arithmetic progression of length $c\delta p^{c\delta^3}$, where $c > 0$ is a small constant. Prove that $A + A + A + A = \mathbb{Z}_p$.

6.3. Let $A \subseteq \mathbb{Z}_p$, $|A| = \delta p$, where δ is a constant. Prove that there exists $x \in A + A + A$ such that

$$0.9\delta|A|^2 < (A * A * A)(x) < 1.1\delta|A|^2.$$

Hint: Use the method of Freiman-Halberstam-Ruzsa.

6.4.* Let $\delta, \varepsilon > 0$ be constants and $A \subseteq \mathbb{Z}_p$. Suppose that each element from $A + A + A$ has at least $\varepsilon|A|^2$ representations in the form $a + a' + a''$. Show that $A + A + A = \mathbb{Z}_p$.