Additive combinatorics

Winter semester 2016/2017

Series V

5.1. Show that $|1 - e^{2\pi i\alpha}| \leqslant 2\pi\|\alpha\|$.

5.2. Let $A, B \subseteq \mathbb{Z}_N$ and $(A+A) \cap B = \emptyset$. Prove that

$$\max_{r \neq 0} |\widehat{B}(r)| \geqslant \frac{|A|}{N}|B|.$$

5.3. Let $A \subseteq \mathbb{Z}_N$. Show that

$$\max_{r \neq 0} |\widehat{(A-A)}(r)| \geqslant \frac{N - |A-A|}{N - |A|}|A|.$$

5.4. Show that if $\mathsf{E}(A) \geqslant |A|^3/K$, then $\mathsf{T}_k(A) \geqslant |A|^{2k-1}/K^{k-1}$.

5.5. Suppose that $A \subseteq [0, N]$ and $A + A = [0, 2N]$. Prove a lower bound for the size of A.

5.6. Let $A \subseteq \mathbb{F}_p$, $|A|t > 100p^{3/2}$. Show that $A \cdot A$ shares an element with every arithmetic progression with length t.

5.7.* Let $A \subseteq \mathbb{F}_p, |A| > p^{3/4}$. Show that $3(A \cdot A) = \mathbb{F}_p$.